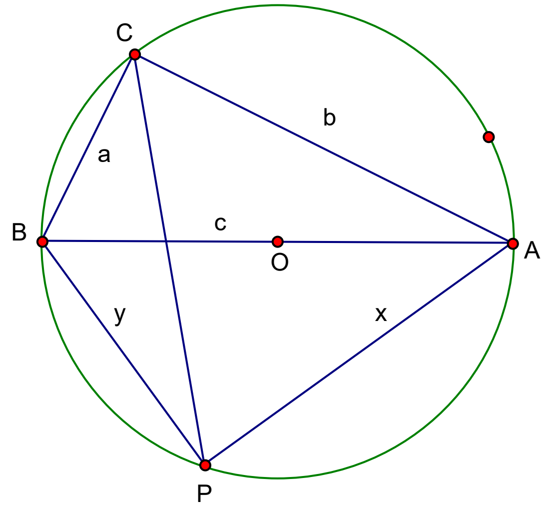
**A point P on the circumcircle of a right-angled Δ**
https://www.linkedin.com/groups/8313943/8313943-6384428834929852419
Find a point P on the circumcircle of a right-angled triangle ABC
($\angle C = 90°$) for which the sum PA + PB + PC is a maximum.
**Solution by Arkady Alt , San Jose, California, USA.**




Since by Ptolemy's Theorem $PC \cdot AB = PA \cdot BC + PB \cdot AC \Leftrightarrow PC \cdot c = x \cdot a + y \cdot b \Leftrightarrow$ $c \cdot PC = ax + by$ then and applying Cauchy Inequality we obtain
$c(PA + PB + PC) = c(x + y) + ax + by =$
$(a + c)x + (b + c)y \leq \sqrt{(a + c)^2 + (b + c)^2}\sqrt{x^2 + y^2} = c\sqrt{(a + c)^2 + (b + c)^2} =$
$c\sqrt{(a + c)^2 + (b + c)^2} = c\sqrt{3c^2 + 2c(a + b)}$.
Equality occurs iff $(x, y) = k(a + c, b + c)$ for some $k > 0 \Leftrightarrow$
$k^2(a + c)^2 + k^2(b + c)^2 = c^2 \Leftrightarrow k = \dfrac{c}{\sqrt{(a + c)^2 + (b + c)^2}} = \dfrac{c}{\sqrt{3c^2 + 2c(a + b)}} = \sqrt{\dfrac{c}{2a + 2b + 3c}}$
that is iff $x = (a + c)\sqrt{\dfrac{c}{2a + 2b + 3c}}$ (then

$$y = \sqrt{c^2 - x^2} = \sqrt{c^2 - \frac{c(a + c)^2}{2a + 2b + 3c}} = (b + c)\sqrt{\frac{c}{2a + 2b + 3c}}).$$

*Thus*, $\max(PA + PB + PC) = \sqrt{3c^2 + 2c(a + b)}$.